

K-out-of-N Functions

Student(s)

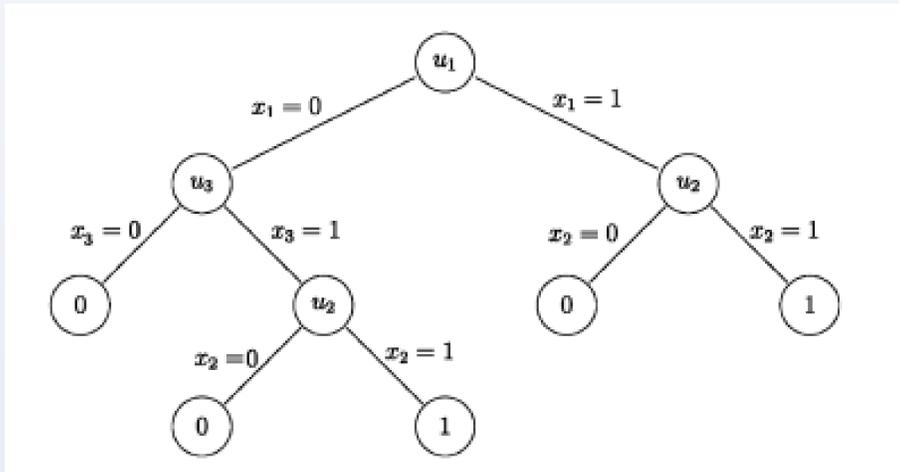
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ABSTRACT



In our research, we addressed the problem of estimating the expected cost of a given strategy for a given k-out-of-n function. The binary decision tree that describes the full strategy has an exponential size in k and n. As a result, using the standard ways over the binary decision tree causes the computation of the strategy's expected cost in exponential time. In the research, we suggest estimating the expected cost by taking samples of the inputs like in the Monte Carlo method. We find out by numerical experiments if the strategy's expected cost can be estimated accurately by this sampling approach.

PURPOSE OF THE PROJECT

The main purpose of the project was writing a code that estimates the expected cost of the optimal strategy that computes k-out-of-n function. In order to assess the quality of the estimation, we compare it with the real expected optimal total cost that can be computed exactly for the special case of k-out-of-n functions [1,2]. This algorithm was coded by Zahed Shahmoradi, a former MSc student at Sabanci University. We analyze the error between the estimated and real values.

ESTIMATING THE COST OF K-OUT-OF-N SYSTEMS WITH MATLAB

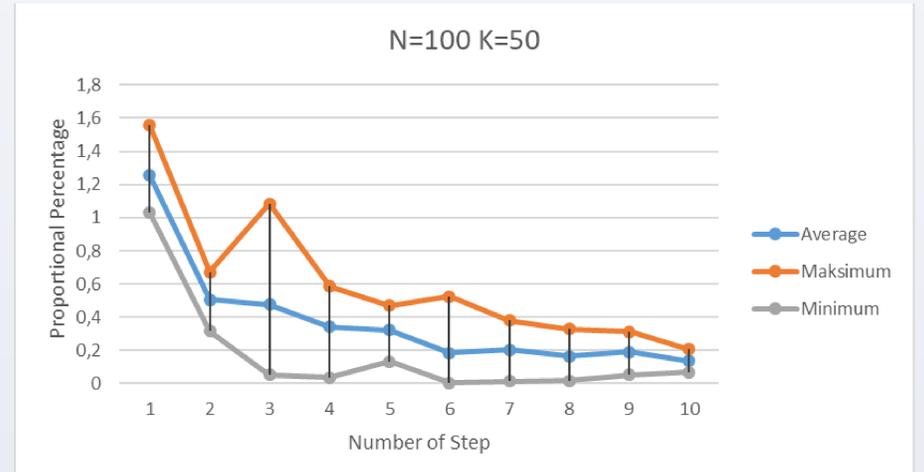
CODE

```
for u=1:Ssiz
    for v=1:NoofComponent
        x=rand(1,1);
        if x <= Probability(v)
            SampVector(u,v)=1;
        else
            SampVector(u,v)=0;
        end
    end
end
```

The sample matrix is built by random distributions of 1s and 0s considering the probability vector. Total average cost is calculated from the sample matrix whose column size is amount of components and row size equals to sample size. Furthermore, matrix of sample size's value changes according to step size. Step size equals to 100 and it checks the total average in each step with using for-loops.

At the end of MATLAB code, aim is to draw a line-graph which includes 3 different lines: **average % difference**, **maximum % difference** and **minimum % difference** of every step out of 5 instances.

COMPARING RESULTS WHEN N IS FIXED AND K IS VARIABLE



In k-out-of-n function, five different data are acquired whose probability and cost values are determined randomly. K-out-of-n's total cost is checked in each of hundred steps and it ends at one thousand which is our sample size value in k-out-of-n function.

On the one hand, the absolute percent deviation is under the 1.6 for maximum, minimum and average values for K=50. Therefore, it shows the minimum error range percentage for all steps at K=50. According to aspects, it is the most stable position for average error.

On the other hand, we obtained the maximum expected optimal cost values at K=40 for all different cost and probability values. At K=90, expected optimal cost values are becoming to minimum for all test case.

Also, convergence of these values is important too. The minimum value of error converges to 0.00012 and the maximum value of error converges to 0.02478. For the average values of error, the value can be accepted as 0.002 except for step size 100. For N=100, we get the most decent and consistent results when K=50. The test data were very approximate to expected optimal cost for it. Also, the graph of K=50 shows that all the 3 lines are proportional to each other in geometric way

To sum up, the minimum error does not exceed 0.5 point except K=20's first two step and K=50's first step. When the sample size increases to 1000, then maximum error gets smaller and smaller for all graphs.

CONCLUSION

In order to assess the quality of the estimation, we conducted numerical experiments. To make a comparison, fixed N=100, and k=10,20,30,40,50,60,70,80,90 are used. For each value of k, we generated 5 independent problem instances.

The found proportion gives us the error of each step. By taking average of all errors, we draw a graph illustrating amount of deviation by hundred intervals. Also, it includes maximum and minimum values. According to results, k-out-of-n function's total cost estimation has 1% absolute percentage difference from the real expected optimal total cost for different K values. In addition, the best fitting sample size is tried to be found by considering to expected optimal value by considering the convergence. A sample size of 1000 seems to be good for the cases that we tried. More experiments needs to be performed for more general results.

REFERENCES

- [1] M. Chang, W. Shi and W.K. Fuchs, Optimal diagnosis procedures for k-out-of-n structures, IEEE Transactions on Computers, 39(4) , 559-564, 1990.
- [2] Y. Ben-Dov, "Optimal testing procedures for special structures of coherent systems", Management Science, 27(12):1410-1420, 1981