Randomness in Complex Analysis & Complex Geometry Workshop

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Statistical Analysis of Zeros of Random Holomorphic Sections

Afrim Bojnik

Sabancı University

In this talk, we will focus on the zero distribution of random holomorphic sections associated with sequences of positive Hermitian holomorphic line bundles on compact Kähler manifolds, as opposed to the traditional setting of tensor powers of a single prequantum line bundle. We present an equidistribution result for systems of random holomorphic sections under a natural convergence assumption on the curvatures. The probability measures considered are subject to a moment condition general enough to include a wide range of commonly encountered measures. Additionally, we discuss an asymptotic normality result for the smooth linear statistics of zeros of Gaussian holomorphic sections. This talk is based on joint work with Ozan Günyüz.

Branching Brownian motion, branching random walks, and the Fisher-KPP equation in spatially random environment

Alexander Drewitz

University of Cologne

Branching Brownian motion, branching random walks, and the F-KPP equation have been the subject of intensive research during the last couple of decades. By means of Feynman-Kac and McKean formulas, the understanding of the maximal particles of the former two Markov processes is related to insights into the position of the front of the solution to the F-KPP equation.

We will discuss some recent result on extensions of the above models to spatially random branching rates and random nonlinearities. Interestingly, the introduction of such inhomogeneities leads to a richer and much more nuanced picture when compared to the homogeneous setting.

Universality limits via canonical systems

Benjamin Eichinger
Vienna University of Technology

It is often expected that the local statistical behavior of eigenvalues of some system depends only on its local properties; for instance, the local distribution of zeros of orthogonal polynomials should depend only on the local properties of the measure of orthogonality. The most commonly studied case is known as bulk universality, where Christoffel-Darboux kernels have a double scaling limit given by the sine kernel. In this talk, I will discuss the first completely local sufficient condition for bulk universality and, much more generally, necessary and sufficient conditions for regularly varying universality limits. The proofs of these results rely on the de Branges theory of canonical systems, and the results also apply to other self-adjoint systems with 2×2 transfer matrices such as Schrödinger operators.
This talk is based on joint works with Milivoje Lukić, Brian Simanek and Harald Woracek.

Zeros of random holomorphic functions on strictly pseudoconvex domains

Hendrik Herrmann
Bergische University of Wuppertal

We consider a relatively compact, smoothly bounded, strictly pseudoconvex domain of dimension greater than one. Let α be a contact form on the boundary and denote by T the CR Toeplitz operator associated to the Reeb vector field and a Reeb invariant volume form. We consider holomorphic functions on the domain arising from holomorphic extensions of random CR functions which are defined on the boundary and are related to eigenvalues of T. As a result, it turns out that their zero set measures concentrate near the boundary in accordance with α when the eigenvalues become large. This is a joint work with Chin-Yu Hsiao, George Marinescu and Wei-Chuan Shen.

Amoeba Measures of Random Plane Curves

Ali Ulaş Özgür Kişisel
Middle East Technical University

In this talk, I will discuss results about the expected areas of amoebas of random complex plane curves, obtained in our joint work with J-Y. Welschinger. It will be shown that the expected area of the amoeba of a degree d plane curve is less than $3\ln(d)^2/2+9\ln(d)+9$, and that the ratio of this expected area to $\ln(d)^2$ is bounded below by $3/4$ as d goes to infinity.

An effective version of Fekete's theorem

Norman Levenberg
Indiana University, Bloomington

A classical result of Fekete gives necessary conditions on a compact set in the complex plane so that it contains infinitely many sets of conjugate algebraic integers. We prove an effective version of Fekete's theorem in terms of a height function. We also give an upper bound on minimal asymptotics of height over sequences of algebraic numbers. This is joint work with Mayuresh Londhe.

Toeplitz operators and zeros of square-integrable Gaussian holomorphic sections

Bingxiao Liu
University of Cologne

For a complete Kähler manifold endowed with a positive line bundle, we use the theory of abstract Wiener spaces to construct a probabilistic model for Berezin-Toeplitz quantization. We associate to a function with compact support (a classical observable) a sequence of square-integrable Gaussian holomorphic sections. Our focus then is on the asymptotic distributions of their zeros in the semi-classical limit, in particular, we prove equidistribution results, large deviation estimates, central limit theorem of the random zeros on the support of the given function. This talk is based on the joint work with Alexander Drewitz and George Marinescu.

Geometric Quantization Results for Semi-positive Line Bundles on a Riemann Surface

Nikhil Savale
University of Cologne

In earlier joint work with G. Marinescu (Math Ann. 2023) we proved the Bergman kernel expansion for semi-positive line bundles over a Riemann surface whose curvature vanishes to at most finite order at each point. In this talk, we explore the related results and consequences of the expansion in the semi-positive case including: Tian's approximation theorem for induced Fubini-Study metrics, leading-order asymptotics and composition for Toeplitz operators, asymptotics of zeroes for random sections, and the asymptotics of holomorphic torsion. The talk is based on recent joint work with G. Marinescu.

Linear statistics of point processes and norm representations

Jordi Marzo
University of Barcelona

A point process is a random locally finite subset of points. Point processes are a powerful mathematical tool appearing in many areas of mathematics (random matrix theory, combinatorics) and in other sciences (forestry, astronomy, epidemiology, wireless networks). Particularly interesting for us are the point processes where there is repulsion between points, as in those given by the zeros of random analytic function or in the, so called, determinantal point processes. In this talk, I will present a new way of studying the asymptotic fluctuations of linear statistics by using norm representations like those proved by Bourgain, Brezis and Mironescu for fractional Sobolev spaces. The talk is based on joint work with Matteo Levi and Joaquim Ortega-Cerdà from Universitat de Barcelona.

Limit theorems for some orthogonal polynomial ensembles

Grzegorz Świderski
Polish Academy of Sciences & University of Wrocław

I will talk on an important subclass of determinantal point processes, called orthogonal polynomial ensembles, which contains some models considered in physics, statistical mechanics, probability theory and combinatorics. Correlation functions for such processes can be expressed in terms of determinants of Christoffel-Darboux kernels known from orthogonal polynomials theory. A popular way of studying such processes is based on analysis of their linear statistics. I will show how asymptotical understanding of Christoffel-Darboux kernels leads to almost sure convergence of properly normalized liner statistics. The talk will be partially based on arXiv:2404.07566.

Weighted holomorphic approximation

Franck Wielonsky
Aix-Marseille University

We consider pairs (G, W), G an open set in $\mathbb{C}$ and W a holomorphic function in G, having the property that any f holomorphic in G can be locally uniformly approximated in G by weighted polynomials of the form $W(z)^n p_n(z),\ deg(p_n) \leq n$. Such pairs were characterized, in terms of potential theory, by Pritsker and Varga in the 90's. Here, we will describe a quantitative version of the result in the spirit of the Bernstein-Walsh theorem. We then consider in more details the special case where G is a loop of a particular lemniscate and $W(z) = 1/(1+z)$, leading to an analog of the famous "Szegő curve" about the zeros of partial sums of e^z. Finally we make a few comments on the multivariate case.
